Developing improved mathematical models of aortic mechanics

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FRAMING THE PROBLEM: PREDICTING RISK OR OCCURRENCE OF ACUTE AORTIC SYNDROMES AND CONSEQUENT INTERVENTION

Despite modern diagnostic technologies and therapeutics, acute aortic syndromes pose substantial mortality and morbidity risks. Aortic operations consequently are distinct with respect to indications in comparison to the majority of cardiovascular surgery in that elective operations are preferentially undertaken based on prophylactic indications. However, identifying which patients and aortas are high risk with respect to dissection and rupture is notoriously difficult and requires a thorough mechanistic understanding of the loading conditions to which the aorta is subjected and the mechanical properties of the aorta. To date, aortic diameter is the principal objective criterion upon which decision making regarding prophylactic aortic operations is based. Aortic diameter at best crudely incorporates both of the factors above.

In summary, there is a clear gap in knowledge and comprehension with respect to aortic mechanics, and thus the quality of decision making is constrained by the quality of information. Current decision making with respect to undertaking prophylactic aortic operations is suboptimal. Moreover, many theoretical and experimental studies, even those that are state of the art in the field, are substantively limited and unable to translate to clinical practice. In this study, in a stepwise fashion, we discuss aortic loading conditions and material properties in reality (as opposed to oversimplifications), the existing theoretical/modeling and some salient experimental literature as well as limitations related to the extant literature, the concept of material inhomogeneity and its importance, and finally moving toward more accurate theoretical models of aortic behavior. We refer the reader to Appendix E1 for a list of terms that we will use and discuss throughout the manuscript, for convenience.

AORTIC LOADING CONDITIONS AND MATERIAL PROPERTIES

Materials undergo motion (including deformation) in response to the application of forces (or loads). Forces are independent variables, whereas motions/deformations are dependent variables. Blood pressure acts normal (perpendicular) and into the aortic intima, whereas blood flow along (major component) and about (minor component) the longitudinal aortic axis induces shear forces parallel to the intimal surface. Force acting on a unit area is traction; the Cauchy stress tensor (σ) at a point relates traction (t) on a surface...
passing through that point and its outward normal (n) through \( t = \sigma n \). \( \sigma \) has 3 normal components, and 6 shear components (of which only 3 are independent due to stress tensor symmetry in the absence of body couples). As a simple representation, if we approximate the aorta (reasonable excepting the aortic root and arterial branch points) to be a cylindrical tube with a circular cross-section and define a cylindrical coordinate system (Figure 1), blood pressure (\( p \)), if blood is assumed to be an incompressible Navier-Stokes fluid, is the mean normal stress (the average of \( \sigma_{rr}, \sigma_{\theta\theta}, \) and \( \sigma_{zz} \) within the blood; Figure 1 depicts intramural stresses). Importantly, the normal stress \( \sigma_{rr} \) acting on the arterial wall is not exactly \( p \), due to arterial wall pulsatile motion, with \( \sigma_{rr} \) involving the partial derivative of the radial component of velocity with respect to the radial coordinate. The shear stresses \( \tau_{rr} \) and \( \tau_{\theta\theta} \) and on the intimal surface are related to shear components of the symmetric part of the velocity gradient, respectively. Aortic wall shear stresses are typically proportional to flow rate. Other than intimal surfaces forces, aortic tethering results in adventitial surface forces. Finally, the aortic root has more complex geometry as well as the presence of a moving valve mechanism, these being additional considerations in this location. The result of all these forces is a fully 3-dimensional state of aortic stress (Figure 1). The effects of gravity are typically modest, and often neglected.

To understand how the aorta moves/deforms under the action of these forces, one needs to find relationships between stresses and deformations. These relations are termed constitutive relations that require the additional knowledge of the material properties in question. For the aorta, these are important features.

- The aorta has a layered structure. Each layer is an inhomogeneous complex mixture of various solid and fluid components. Moreover, blood, as the medium directly exerting loads upon the aorta by virtue of systemic ventricular action upon it, is typically modeled as a homogeneous non-Newtonian fluid (although it may be conceived as a Navier-Stokes (Newtonian) fluid in large arteries such as the aorta).
- The aorta is anisotropic, with the anisotropy changing as a function of location, and this change is patient-specific. This spatial variation of anisotropy is because various components of the mixture (different cell types and extracellular matrix proteins) comprising the aorta have different anisotropies and cannot inhabit the same exact location. That is, within small spatial subdivisions only a cell or an extracellular matrix component—each of a particular type—can be present, and thus because different spatial locations have structurally and functionally different components, anisotropy necessarily exists. Such would be the case even in a homogenized representation of the aorta, because the homogenized densities of the components with different anisotropies will be different at different locations. That this is patient-specific is again almost certain, just as almost all physiological variables have interindividual variation.
- The response characteristics of the different constituents comprising the aorta are different. Some components are elastic, some are viscoelastic, and others are fluid.
- Although described as incompressible, the aortic wall is slightly compressible.
- The response of most of the constituents is nonlinear.

**FIGURE 1.** A depiction of the components of the Cauchy stress tensor on a cylindrical object, in cylindrical coordinates. A segment of the aortic wall is depicted. The normal stresses are denoted as \( \sigma \), with **solid lines**, \( \sigma_{rr} \) is the radial normal stress acting through the aortic wall from the inner to the outer surface of the aorta (blue squares indicate inner and outer surfaces) and thus along a radius from the centerline. It is always orthogonal to both (1) the circumferential tangent line (for a circular cross-section, this line will be tangential to the curvature of the wall) and (2) the axial tangent line (for a rectangular cross-section, this line will be axially oriented). The other 2 normal stresses act along these lines, \( \sigma_{\theta\theta} \) is the circumferential “hoop” normal stress acting through the circumferential wall thickness (red squares) and along the line instantaneously tangential to the curvature of the wall. \( \sigma_{zz} \) is the axial normal stress acting through the axial wall thickness (green squares) and along the length of the aorta. The shear stresses are denoted as \( \tau \) (**dashed lines**). These act along (parallel to) rather than through (perpendicular to) the aortic surfaces. (Modified from Rajagopal,1)
PREVIOUS EXPERIMENTAL AND THEORETICAL WORK, AND IDENTIFYING AREAS FOR IMPROVEMENT

Wertheim\textsuperscript{4} seems to have been the first to systematically carry out mechanical experiments on different human tissues, including blood vessels. He described the data for blood vessels by fitting the constitutive relation

\[ \varepsilon^2 = A\sigma^2 + B\sigma, \]  

(1)

where \( \varepsilon \) is the longitudinal strain, \( \sigma \) is the longitudinal stress, and \( A \) and \( B \) are constants. Wertheim\textsuperscript{4} provided values for \( A \) and \( B \) for the different materials upon which he experimented. The constitutive relation\textsuperscript{7} describes the response of a nonlinear elastic body and does not recognize that most of the materials that Wertheim experimented on were viscoelastic and he also used the linearized strain. The reason that his data fit an elastic relationship was that he did not allow the tested materials to undergo either creep (increase in strain over time under constant stress) or stress relaxation (decrease in stress over time under constant strain), each of which is a viscoelastic property. Wertheim\textsuperscript{4}’s work was followed by others in the second half of the 19th century. Weber,\textsuperscript{5} and then Roy,\textsuperscript{6} described creep in animal tissues. Roy\textsuperscript{6} thus recognized that animal tissues are viscoelastic, and he refers to the viscosity of animal tissues. Interestingly, despite such early studies recognizing the viscoelasticity the arterial wall, practically all the constitutive relations used to describe arteries assume that the arterial wall is elastic.

Constitutive theories that assume blood vessels to be elastic are either excessively reductionist, like the constitutive relation for a linearized elastic solid that cannot accomplish the task of describing the behavior of blood vessels when subject to physiologically relevant loading conditions, or they are more complex but yet omit critical material features. The studies that assume blood vessels to be nonlinearly elastic are too numerous to document and discuss (we refer readers to Humphrey\textsuperscript{7} and Myneni and Rajagopal\textsuperscript{8}). In a departure from other nonlinear elastic constitutive relations, Lanir\textsuperscript{9} developed a relation wherein the volume fraction of the constituents, fiber orientation density distribution, and waviness of the fibers, are incorporated. However, these quantities are patient-specific and require information on individual aortic microstructure, which would be formidable in vivo (see Myneni and Rajagopal\textsuperscript{8}).

Most of the early theoretical studies modeling the viscoelastic response of arteries adopted the theory of linearized viscoelasticity (see Lawton,\textsuperscript{10} Bergel,\textsuperscript{11} and Westerhof and colleagues\textsuperscript{12}). The aortic wall undergoes large deformations where the use of linearized theory is inappropriate (see below for further discussion).

More recently, our own group’s developed a model of acute aortic dissection,\textsuperscript{13} which we now review. The aorta was modeled as either transversely isotropic or orthotropic layered viscoelastic body. In contrast to the previously discussed studies, this model is more mechanically appropriate and is also more mathematically/theoretically sound in our view for the study of aortic dissection. However, this work too has 2 important simplifications that may not be appropriate. In this study, inhomogeneity was due to layered structure, but within a layer, we assumed that the aorta is homogeneous. This is certainly an oversimplification because even grossly evident aortic layers have differing constituents within them—cells of different types and extracellular matrix components. Appealing to an integral constitutive relation for nonlinear viscoelastic response that belongs to the class of constitutive relations developed by Pipkin and Rogers,\textsuperscript{14} assuming that the body is layered and that the anisotropy of each layer is the same, namely transversely isotropic or orthotropic, and using the representation derived by Rajagopal and Wineman,\textsuperscript{15} we put into place an integral constitutive relation. Assuming that each layer of the aorta was homogeneous, dissection initiation was studied based on shear stress reaching a certain critical value. As discussed in detail later, using stress as the only criterion for dissection is inappropriate even in our own view; this is a second but nonessential oversimplification.

Finally and most recently, studies have employed either linearized elastic,\textsuperscript{16} unclear,\textsuperscript{17} or better exponential-type hyperelastic\textsuperscript{18} models of the aorta in analyzing experimental data. Although the experimental approaches are of high quality, the theoretical models employed to analyze the experimental data are not suited to the data. For example, in Vianna and colleagues,\textsuperscript{16} although the experimental data are nonlinear, a linearized theoretical model is employed. Similarly, in Eliathamby and colleagues\textsuperscript{17} the authors conducted experiments in which aortic samples are subjected to equibiaxial strains of 25%, with assessments of so-called tangent moduli at strains of 10%. However, anything beyond \( \sim 5\% \) is considered to be a large deformation. Certain assumptions and analysis procedures used in this work are unsuitable for nonlinear, anisotropic, viscoelastic bodies like the aorta. Such errors are not limited to these studies, most notable in Eliathamby and colleagues,\textsuperscript{17} but appear pervasive in the experimental literature. We highlight notable ones from Eliathamby and colleagues\textsuperscript{17} because this study in our view has the most limitations.

- The authors define the ratio of the change in length to the original length, also known as the extensional strain, to be the strain in the material. For example, the “extensional strain” \( \gamma_{i} \), in the \( i^{th} \) direction is related to the nonlinear Green-St. Venant strain \( E_{ii} \), in the \( i^{th} \) direction, through \( \gamma_{i} = (1+2E_{ii})^{\frac{1}{2}} - 1 \), and equals \( E_{ii} \), the nonlinear strain, only when the strain is very small (\( E_{ii} \ll 1 \)). This is true if one uses the Almansi-Hamel strain or any other proper
nonlinear measure of strain. However, the strains in the authors’ study are not small. Without realizing or acknowledging it, what the authors use is a linearized measure of strain.

- For a general nonlinear anisotropic elastic body, there is nothing that can be called a modulus of elasticity. There are several material moduli, especially for nonlinear anisotropic elastic bodies, and depending on the constitutive relation, different material moduli will come into play in a biaxial experiment. Furthermore, although the authors conducted biaxial experiments and defined a tangent modulus, the notion of a tangent modulus only applies to 1-dimensional response (uniaxial loading) of a material. The concept has no meaning in a general deformation, or a bi-axial deformation.

- In viscoelastic materials, which their specimens are, as they clearly exhibit hysteresis, the stress depends on both the strain and the strain rate, and thus, there is nothing called a stress–strain curve, only that one can have a relationship between the stress and the strain at a fixed strain rate. There are an infinite number of stress–strain curves, each for a different strain rate.

- Even if one is interested in capturing the underlying elastic response for a viscoelastic body, one has to subject it to instantaneous loading to determine the elastic response, which is clearly not the case in their experiments.

- For tissues, among the main difficulties is measuring a reference state; that is, a stress-free or strain-free configuration, from which stresses and strains can be measured. The form of the constitutive relation will change with the reference configuration from which quantities like strain are measured.

In addition, with respect to this and the other recent studies, the commentaries associated with the studies have identified and discussed several of their substantive limitations.19-21

THE CONCEPT OF INHOMOGENEITY AS IT APPLIES TO THE AORTIC WALL

Motivating Argument and Conjecture

Inhomogeneity within the aortic wall has not previously been considered to be of significance to acute aortic syndromes, although our previous work has incorporated it partially, vis-à-vis interlaminar inhomogeneity. We now introduce more broadly the concept of inhomogeneity. A body $\mathcal{B}$ is said to be inhomogeneous if different particles (represented mathematically by points) that belong to the body respond differently. Experimentally, mathematical points cannot be assessed. We have to apply the notion of inhomogeneity to sufficiently small chunks that belong to the body as having different properties. Density and crystallographic structure (material symmetry group) are properties that can change from point to point. For the aorta, the fact that it is composed of several different components with different properties makes it abundantly clear that it is inhomogeneous. The properties of collagens, elastin, and smooth muscle cells are different, and at any point, because only 1 component can exist, the properties of the various points in the aorta will vary.

We propose that acute aortic syndromes are largely a consequence of the inhomogeneity of the aorta, with additional contributions of other aortic material properties and aortic loading conditions; this is based on experience with numerous other materials that have been studied. If this is so, then the typical approach of using stress or strain as a criterion for failure (which even our previous work13 employed) is untenable because one does not know what is termed the initial stress or strain in the body. This is particularly relevant to blood vessels because one does not know a priori the stress-free or strain-free configuration. Recently, it has been suggested that failure such as tearing, rupture, and cavitation occur in inhomogeneous elastic and viscoelastic solids at locations of lowest density in the body, unless there are extreme stress concentration factors. This has been found to hold true in several specific boundary value problems for a class of inhomogeneous elastic and viscoelastic bodies the results of Alagappan and colleagues22-24 agree well with the experimental results on rupture in rubber in Gent and Linley,25 and the study by Karunakaran and colleagues26 agrees with the initiation of damage for elastomers with weld lines (see E2). The approach also works in materials like concrete as evidenced by agreement with experiments in the studies of failure of concrete due to purely mechanical loading (see E2) and due to chemical-mechanical loading of concrete (see E2).

Experiments on Polydimethylsiloxane to Test the Conjecture

To test this hypothesis, we constructed simple specimens of materials with inhomogeneity using polydimethylsiloxane (PDMS) of 2 different densities within each specimen. PDMS is an organosilicon polymer that has been used as an experimental model system for the aorta based on its nonlinear and viscoelastic properties (E5-E7). The samples were such that in 1 set the material with the lower density was at the center of the specimen, and in the second set, the lower density material was near the 2 grips toward the ends of the specimen. In both cases, breakage occurred in the region where the density was lower (Figure 2).

Possible Methodology for Measuring Aortic Wall Density

Although the value of measuring physical densities of tissues using noninvasive imaging techniques such as magnetic resonance imaging (MRI) or radiograph computed tomography (CT) has long been recognized, the practical
realization of this idea has remained a challenging problem. At a physical level, the relationship between the material density of tissues and the contrast mechanisms underlying the imaging modality such as MRI or CT is not direct, and at a biological level, tissue composition can vary substantially not only between normal and pathologic states of tissue, but also even among healthy individuals due to factors such as under/overnutrition, age (infant vs adult), or physical activity (E8) that renders realistic modeling of in vivo tissue composition challenging.

Nonetheless, representation of tissue radiograph attenuation coefficient ($\mu$) in terms of electron density ($nZ$, where $n$ is the number of atoms per unit volume, $Z$ is the atomic number), and photon energy $E$ have been proposed for elements and mixtures (for a detailed description seeE9). This approach has been extended to derive a relationship between the CT number (in Hounsfield units) of a given voxel (a cubic graphic unit) to relative electron density (rED) (ie, density relative to water) using calibration curves experimentally obtained from a set of tissue-mimicking reference materials of known electron and mass densities (E10). The advent of dual-energy CT and multi-energy CT technologies has opened several new avenues to determine rED and material composition from 2 or more CT numbers of a given voxel ($E^{11}$). It should be noted that the spatial resolution attainable in most clinically available scanners is $\sim 0.125$ mm$^3$ to 1.0 mm$^3$—a volume that is several orders of magnitude compared with the elemental composition within the voxel. Despite this relatively large voxel size, these approaches can be refined further to estimate the relative variation of rED across the vessel wall. Furthermore, complementary imaging approaches such as MRI could provide valuable soft-tissue information across the vessel wall as well as provide information about time-resolved, 3-dimensional blood flow velocities across the aorta without incurring radiation burden or nephrotoxic contrast media burden. We speculate that combining coregistered tomographic information from MRI and CT could yield information about regional heterogeneity in tissue density across the aortic wall.

**DEVELOPMENT OF A NEW CONSTITUTIVE RELATION INCORPORATING A THERMODYNAMIC FRAMEWORK WITH MAXIMIZATION OF ENTROPY PRODUCTION AND EVOLUTION OF THE NATURAL CONFIGURATION**

The drawbacks of using integral-type viscoelastic models such as the 1 introduced in Rajagopal and colleaguesE13 were discussed in detail in Myneni and RajagopalE8 As an advance from our previous theoretical work, which was based on an integral-type viscoelastic model, a rate-type

<table>
<thead>
<tr>
<th>Group</th>
<th>Number of Samples</th>
<th>Thickness (mm)</th>
<th>Max Stress at Break (kPa)</th>
<th>Stretch at Break</th>
</tr>
</thead>
<tbody>
<tr>
<td>Higher density in center (right)</td>
<td>10</td>
<td>1.42 ± 0.16</td>
<td>463 ± 50.0</td>
<td>2.91 ± 0.16</td>
</tr>
<tr>
<td>Higher density near grips (left)</td>
<td>10</td>
<td>1.37 ± 0.35</td>
<td>666 ± 107</td>
<td>2.99 ± 0.19</td>
</tr>
</tbody>
</table>

**FIGURE 2.** Polydimethylsiloxane (PDMS) samples were prepared using a curing agent in ratios of 10:1 and 20:1 by weight. PDMS is translucent; therefore, before mixing the prepolymer solutions, a specific dye was placed into each of the 2 different ratios. The higher density ratio was red, and the lower density was yellow.
A constitutive relation to describe an anisotropic inhomogeneous viscoelastic material would be appropriate to describe aortic behavior. A thermodynamic basis can be provided for the constitutive relation by appealing to the general thermodynamic framework, which is based on the notion that the natural configuration of a body evolves as it undergoes a thermodynamic process, and this configurational evolution is determined by maximizing the rate of entropy production. Such a procedure has been successful for numerous classes of materials (viscoelastic fluids \((E12)\), classical plasticity \((E13)\), twinning and solid-to-solid phase transition \((E14)\), asphalt \((E15)\), crystallization of polymers \((E16)\), shape memory polymers \((E17)\), chemically reacting mixtures \((E18)\), granular materials \((E19)\), wet granular solids \((E20)\), and other materials).

A thermodynamic process satisfies the balance of mass, linear momentum, and energy are

\[
\frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{v}) = 0, \tag{2}
\]

\[
\rho \frac{d\mathbf{v}}{dt} = \text{div}\sigma + \rho \mathbf{b}, \tag{3}
\]

\[
\rho \frac{d\mathbf{e}}{dt} = \sigma \mathbf{L} - \text{div}\mathbf{q} + \rho r, \tag{4}
\]

and the second law of thermodynamics in the form

\[
\sigma \mathbf{L} - \rho \frac{d\mathbf{e}}{dt} + \rho \mathbf{b} \frac{d\mathbf{\eta}}{dt} \quad \mathbf{\theta} \frac{\text{grad}\mathbf{\theta}}{\theta} = \rho \theta \zeta := \zeta \geq 0, \tag{5}
\]

In the above equation, \(\rho\) is the density, \(\mathbf{v}\) is the velocity, \(\sigma\) is the Cauchy stress (that is assumed to be symmetric), \(\mathbf{b}\) is the specific body force, \(\mathbf{q}\) is the heat flux, \(r\) is the radiant heating, \(\theta\) is the temperature, and \(\zeta\) is the rate of entropy production. In equation (5), if we refer to \(\zeta\), which is the sum of all of the various rates of entropy production (dissipation, heat conduction, mixing, phase change, and so on), as the "generalized dissipation," then maximization of the total rate of dissipation has to be performed.

Within a purely mechanical context, the rate of dissipation is given by

\[
\zeta = \sigma \mathbf{L} - \rho \frac{dW}{dt}. \tag{6}
\]

The constitutive relation arises as a consequence of maximizing the rate of dissipation with Eq 6 being used as a constraint; that is, we maximize

\[
\Phi = \zeta + \lambda \left( \zeta - \sigma \mathbf{L} - \rho \frac{dW}{dt} \right). \tag{7}
\]

Within the context of a purely mechanical perspective, for the aorta, we assume that the stored energy \(W\) for the material is given by

\[
W = W(\rho, C), \tag{8}
\]

where \(C\) is the right Cauchy-Green tensor. In the case of an orthotropic material, \(W\) will depend on the appropriate invariants of \(C\).

The rate of dissipation \(\zeta\) is assumed to be

\[
\zeta = \zeta(B, D), \tag{9}
\]

where \(B\) is the left Cauchy-Green tensor and \(D\) is the symmetric part of the velocity gradient. We shall not develop a specific constitutive relation here. The procedure to be followed is precisely the same as that used in the above-mentioned articles by making specific assumptions for the manner in which energy is stored and how it is dissipated.

CONCLUSIONS

At present, we lack the ability to reliably predict the occurrence, or even approximate risks, of the occurrence of acute aortic syndromes. To do so requires a substantially improved understanding of aortic mechanics—both aortic loading conditions and aortic mechanical properties—brought together to formulate accurate and precise theoretical models. However, previous work has failed to consider true and important aspects of aortic mechanics, limiting both its explanatory power and predictive capacity. In this study, we have discussed the essential features of a sound model of aortic mechanics, introduced the hypothesized importance of aortic wall inhomogeneities as areas of tissue failure, suggested methodological approaches to ascertain inhomogeneity, and introduced preliminary elements of an improved mathematical model of aortic mechanics. A substantial amount of further work will be required to advance both theoretical understanding and experimental study.

Conflict of Interest Statement

The authors reported no conflicts of interest.

The Journal policy requires editors and reviewers to disclose conflicts of interest and to decline handling or reviewing manuscripts for which they may have a conflict of interest. The editors and reviewers of this article have no conflicts of interest.

References


Key Words: acute aortic syndromes, rupture risk, aortic dissection, mass density, inhomogeneity, constitutive relations, arterial mechanics
APPENDIX E1. DEFINITIONS

Non-Newtonian Fluids
Fluids that cannot be described by the classical Navier-Stokes constitutive relation. The most common condition that non-Newtonian fluids do not satisfy is constancy of viscosity with shear rate, although there are several others.

Anisotropy
Directional dependence of a property of a physical entity. In the context of material behavior, this means directional dependence of material properties. For a discussion concerning simple materials, see Truesdell and Noll (E21), for materials related by implicit relations, see Rajagopal (E22).

Viscoelasticity
Viscoelastic body refers to a body that is essentially an elastic body exhibiting some viscous characteristics as opposed to an elasticoviscous body that is essentially a viscous body that also exhibits some elastic characteristics. A viscoelastic body exhibits instantaneous elasticity and stress relaxation, whereas an elasticoviscous body cannot exhibit instantaneous elasticity or stress relaxation. Both viscoelastic bodies and elasticoviscous bodies can exhibit shear thinning/shear thickening, creep, and normal stress differences in simple shear flow.

Inhomogeneity
The concept of inhomogeneity refers to different particles of the abstract set of particles comprising the abstract body responding differently to external stimuli. The fact that particles belonging to a specific deformed configuration of the body respond differently to external stimuli does not imply that the body is inhomogeneous. A body is said to be homogeneous if the abstract body is such that the response of all the particles belonging to it are the same. A rigorous mathematical definition of inhomogeneity can be found in Noll (E23), and Truesdell and Noll (E24).

Body
A set, whose members are particles, on which one can define the notion of mass (measure) and a topology (open sets).

Point
Whereas the notion of a point is assumed to be an axiom and thus intuitively obvious, it is far from being so. See Rajagopal (E24) for a detailed discussion of the notions of particles and points.

Material Symmetry Group
The set of linear transformations (that form a group) under which the response functions are invariant (the response remains the same under the same applied stimuli).

Natural Configuration
The configuration that the body takes on the removal of the external stimuli.

Rate-Type Constitutive Relation
A constitutive relation between: (1) the stress and the time rates of stress, (2) appropriate kinematical quantities and their time rates, (3) temperature and its time rate, (4) density and its time rate, and so on.

Invariants
Properties of linear transformations (colloquially matrices) that do not change with the basis that is used.

E-References